Geodesy 1 (GED203)

Lecture No: 10

STRENGTH OF FIGURE & SATELLITE STATION

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Overview of Previous lecture

TRIGONOMETRIC LEVELING

TRIGONOMETRIC LEVELING - OBSERVATION METHODS

TRIGONOMETRIC LEVELING – CORRECTIONS

TRIGONOMETRIC LEVELING – NUMERICAL EXERCISE

PRECISE LEVELING

PRECISE LEVELING - EQUIPMENT

APPLICATIONS OF PRECISE LEVELING

OVERVIEW OF TODAY'S LECTURE

STRENGTH OF FIGURE

COMPUTATION OF STRENGTH OF FIGURE

NUMERICAL EXERCISES ON STRENGTH OF FIGURE

SATELLITE STATION PROBLEM

SIGNIFICANCE OF SATELLITE STATION

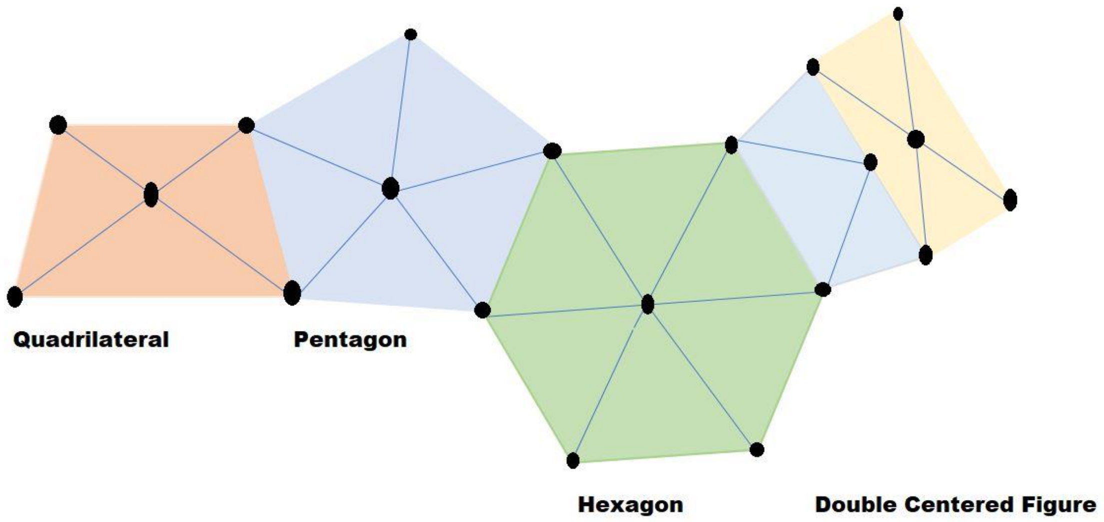
REDUCTION TO CENTER

NUMERICAL EXERCISES ON SATELLITE STATION

Expected Learning Outcomes

1. Understanding the concept of strength of figure in triangulation networks.
2. Learning about the factors that contribute to the strength of a figure such as geometry.
3. Understanding the fundamental equations and mathematical models used to compute the strength of figures.
4. Understanding the concept of satellite stations and reduction to center.
5. Understanding the impact of satellite stations on the establishment of geodetic control.

(1) Strength of Figure



Strength of Figure

- A measure of the judicious selection of the framework consisting of triangles and quadliterals employed for triangulation.
- Determination of the figure which gives the least error in calculated length of last line in the system due to the shape of triangles and computation of the figures.

$$R = F\sum(\delta_A^2 + \delta_A\delta_B + \delta_B^2)$$

R: Strength of Figure

F: Strength of Figure Factor $F = \frac{D-C}{D}$

$\delta_A \delta_B$: The logarithmic differences corresponding to 1" for distance angles A and B

Strength of Figure

- **F: Strength of Figure Factor F =** $\boldsymbol{\frac{D-C}{D}}$

D: total number of observed directions except the base line

C: total number of conditions $C = (L' - S' + 1) + (L - 2S + 3)$

L': number of lines observed in both directions including the baseline.

S': number of occupied stations

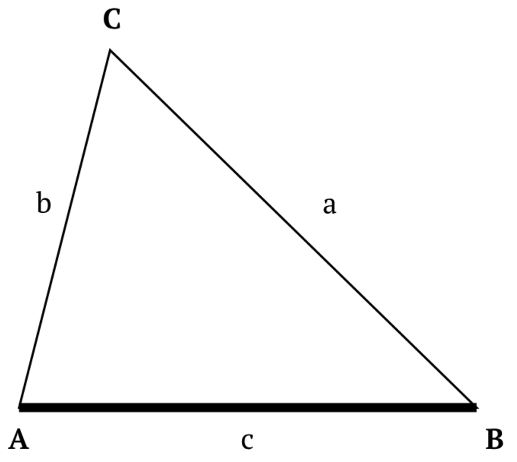
L: total number of lines

S: total number of stations

STRENGTH OF FIGURE

- In a triangulation network, all angles are observed and a base line while the lengths of other lines are computed based on sine rule.

$$\frac{AB}{\sin C} = \frac{BC}{\sin A} = \frac{AC}{\sin B}$$



$$\therefore BC = \frac{AB \sin A}{\sin C}$$

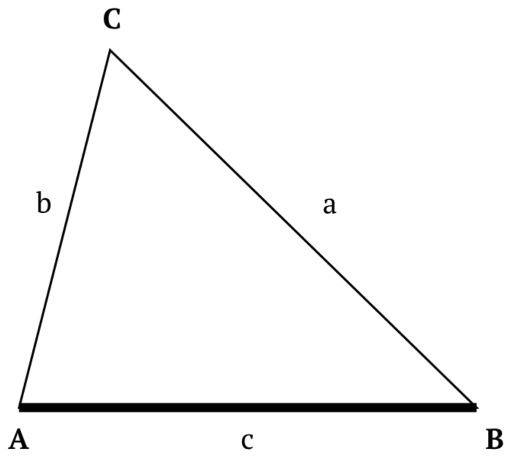
$$\therefore AC = \frac{AB \sin B}{\sin C}$$

- How much a side length is affected if an angle contain an error of 1 arcsecond?

Strength of Figure

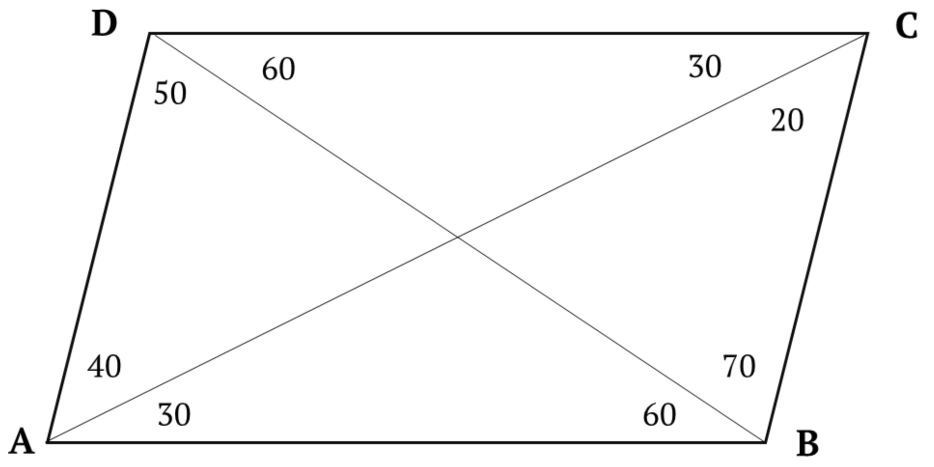
- How much a side length is affected if an angle contain an error of 1 arcsecond?

$$\text{Difference} = \log\sin A - \log\sin(A + 1'') = 2.1 \cot A$$



Strength of Figure – Numerical Exercise

- **(1) Determine the best route to calculate the side CD from the known side AB in the shown figure.**



$L' = 6, L = 6, S = 4\ , and\ S' = 4$

$C = (L' - S' + 1) + (L - 2S + 3) = (6 – 4 + 1) + (6 – 8 + 3) = 4$

$D = 10$

$$\boldsymbol{F = \frac{D - C}{D} = \frac{10 - 4}{10} = 0.60}$$

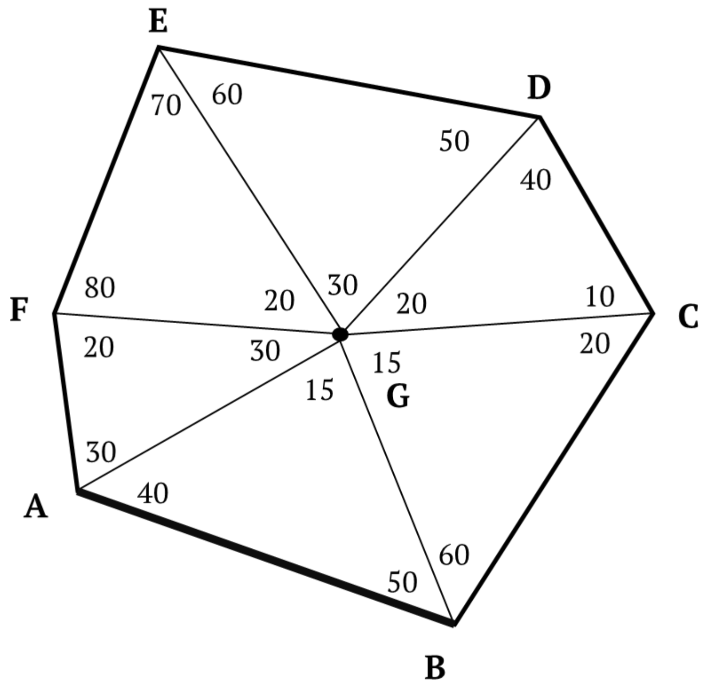
Strength of Figure – Numerical Exercise

Route No	Triangle	Known Side	Unknown Side	Distance Angle A	Distance Angle B	$(\delta_A^2 + \delta_A\delta_B + \delta_B^2)$	$\sum(\delta_A^2 + \delta_A\delta_B + \delta_B^2)$	R
R_1	ABC	AB	BC	20	30	67.505	70.49	42.29
	BCD	BC	CD	60	70	2.98		
R_2	ABD	AB	BD	50	70	5.036	10.07	6.04
	BDC	BD	DC	50	70	5.036		
R_3	BAC	BA	AC	20	130	26.227	31.16	18.7
	ACD	AC	CD	110	40	4.934		
R_4	BAD	BA	AD	50	60	6.711	35.31	21.18
	ADC	AD	DC	30	40	28.596		

Then, the best route to compute side CD from the baseline AB is $\mathbf{R_2}$ using the triangles **ABD** and **BDC**.

Strength of Figure – Numerical Exercise

- **(2) Determine the best route to calculate the side ED from the known side AB in the shown figure.**



$L' = 12, L = 12, S = 7\ , and\ S' = 7$

$C = (L' - S' + 1) + (L - 2S + 3)$ = (12 – 7+ 1) + (12 – 14 +3) = 7

$D = 22$

$$\boldsymbol{F = \frac{D - C}{D} = \frac{22 - 7}{22} = 0.68}$$

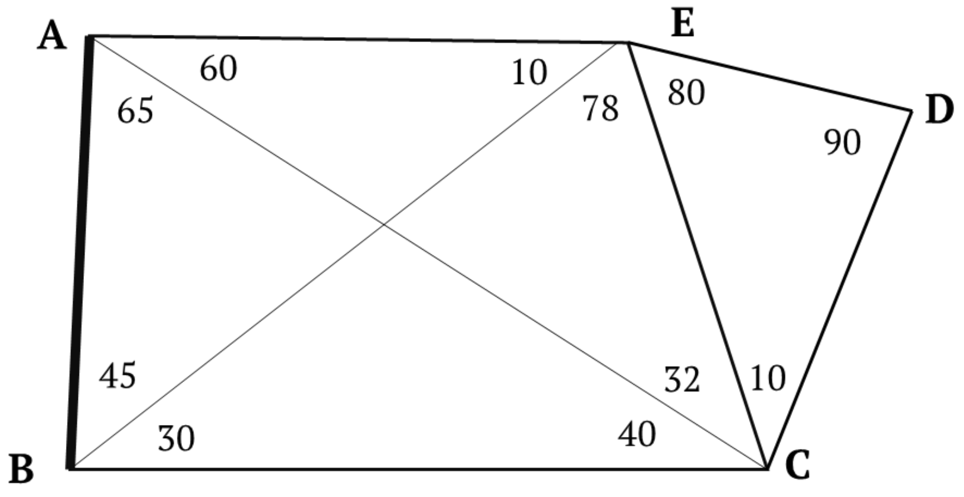
Strength of Figure – Numerical Exercise

Route No	Triangle	Known Side	Unknown Side	Distance Angle		$(\delta_A^2 + \delta_A\delta_B + \delta_B^2)$	$\sum(\delta_A^2 + \delta_A\delta_B + \delta_B^2)$	R
				A	B			
R_1	ABG	AB	BG	15	40	87.3	326.1×0.68	221.7
	GBC	BG	GC	20	60	41.8		
	GCD	GC	GD	40	10	177.9		
	GDE	GD	ED	60	30	19.1		
R_2	ABG	AB	AG	15	50	78.3	169.6×0.68	115.3
	AGF	AG	GF	20	30	67.5		
	GFE	GF	EG	70	80	1.0		
	EGD	EG	ED	50	30	22.7		

Then, the best route to compute side ED from the baseline AB is $\mathbf{R_2}$ using the triangles **ABG, AGF, GFE**, and **EGD**

Strength of Figure – Numerical Exercise

- **(3) Determine the best route to calculate the side ED from the known side AB in the shown figure.**



$L' = 8, L = 8, S = 5\ , and\ S' = 5$

$C = (L' - S' + 1) + (L - 2S + 3) = (8 - 5 + 1) + (8 - 10 + 3) = 5$

$D = 14$

$$\boldsymbol{F = \frac{D - C}{D} = \frac{14 - 5}{14} = 0.64}$$

STRENGTH OF FIGURE – NUMERICAL EXERCISE

Route No	Triangle	Known Side	Unknown Side	Distance Angle A	Distance Angle B	$(\delta_A^2 + \delta_A\delta_B + \delta_B^2)$	$\sum(\delta_A^2 + \delta_A\delta_B + \delta_B^2)$	R
R_1	ABC	AB	BC	40	65	9.7	166.6	106.6
	BCE	BC	EC	78	30	15.1		
	CED	EC	ED	90	10	141.85		
R_2	ABC	AB	AC	40	75	7.98	151.4	96.89
	ACE	AC	EC	83	60	1.6		
	CED	EC	ED	90	10	141.85		
R_3	ABE	AB	AE	10	45	171.3	329.95	211.2
	AEC	AE	EC	32	60	16.8		
	CED	EC	ED	90	10	141.8		
R_4	ABE	AB	BE	10	125	126.5	284.5	182.1
	BEC	BE	EC	72	30	16.18		
	CED	EC	ED	90	10	141.85		

Then, the best route to compute side ED from the baseline AB is $\mathbf{R_2}$ using the triangles **ABC, ACE,** and **CED**

(2) Reduction to Center (Satellite Station)

Satellite Station – Problem Definition

- In a triangulation network, mosques, church spires, or any similar tall objects are marked as triangulation stations.
- Such types of stations cannot be occupied.
- Consequently, the instrument is set up on an auxiliary station which is so close to the main station.
- This auxiliary station is called "**Satellite Station**"
- At the satellite station, all angles to the adjacent stations are measured with the same precision of other angles in the system.
- The process of computing the main angle at the inaccessible station from the measured ones is called "**Reduction to center**"

Satellite Station – Problem Definition

- From the shown figure:

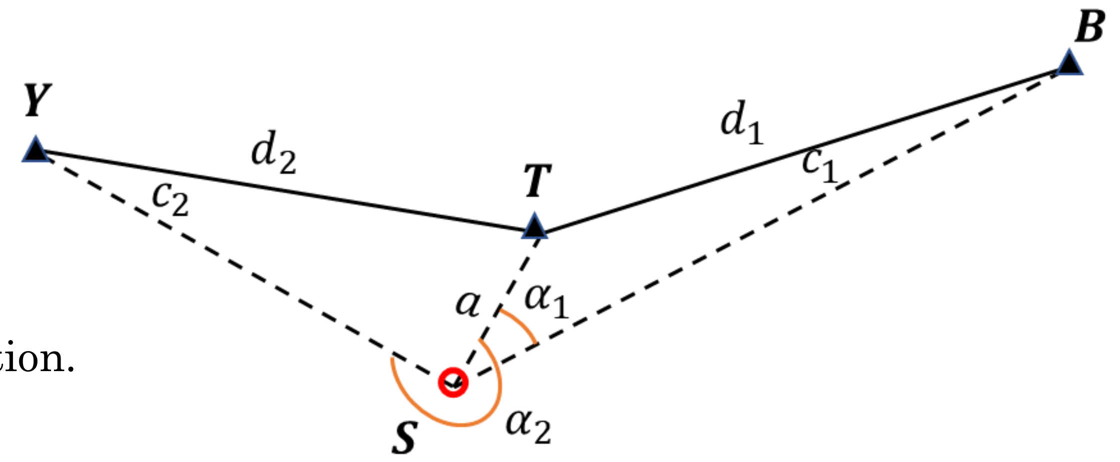
Y, T, and ***B***: triangulation stations

T: inaccessible triangulation station

S: satellite station

α_1 , α_2: observed angles

a : distance between satellite station and inaccessible station.



By solving triangles TSB, and TSY

$\frac{\sin c_1}{a} = \frac{\sin \alpha_1}{d_1}$, and $\frac{\sin c_2}{a} = \frac{\sin \alpha_2}{d_2}$

i.e.,

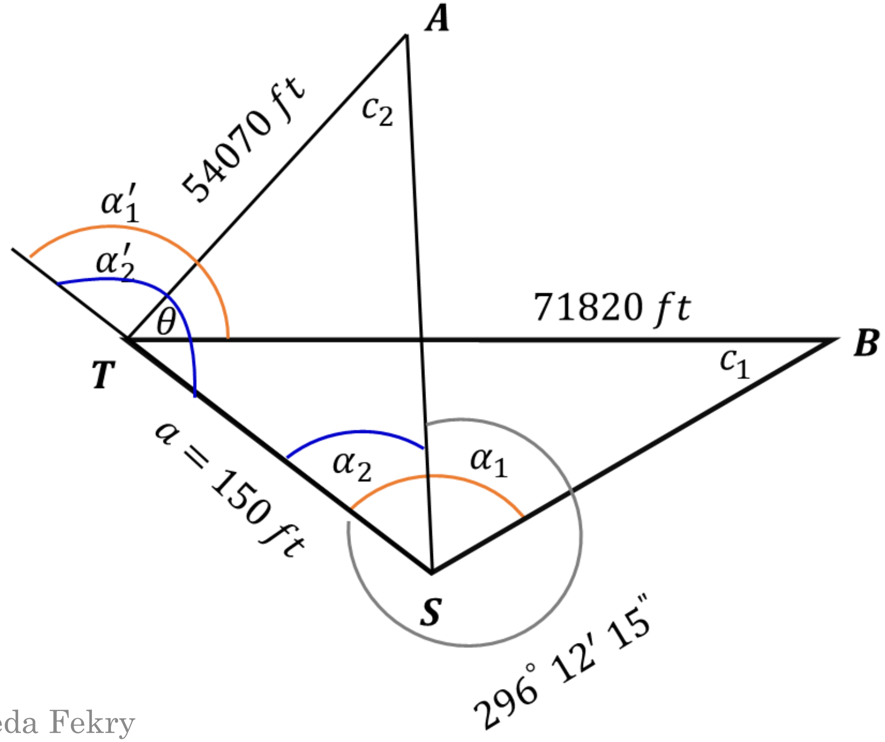
$c_1 = sin^{-1} \frac{\text{a}\times\sin \alpha_1}{d_1\times\sin 1''}$

$c_2 = sin^{-1} \frac{\text{a}\times\sin \alpha_2}{d_2\times\sin 1''}$

Such that c_1, and c_2 are given in seconds.

Satellite Station – Numerical Exercise

➢ (1) Directions were observed from a satellite station S, 150 ft apart from the main triangulation station T such that direction SA = $00^\circ\ 00'\ 00''$, SB = $71^\circ\ 54'\ 30''$, and ST = $296^\circ\ 12'\ 15''$. Compute the subtended angle (ATB) at the main station T if the length of sides TA = 54070 ft, and TB = 71280 ft.



$$\alpha_2 = 360 - 296^\circ\ 12'\ 15'' = 63^\circ\ 47'\ 45''$$

$$\alpha_1 = \alpha_2 + \widehat{ASB} = 63^\circ\ 47'\ 45'' + 71^\circ\ 54'\ 30'' = 135^\circ\ 42'\ 15''$$

$$c_1 = sin^{-1}\frac{\text{a}\times\sin\alpha_1}{d_1\times\sin 1''} = sin^{-1}\frac{150\times\sin 135^\circ\ 42'\ 15''}{71280\times\sin 1''} = 303''$$

$$c_2 = sin^{-1}\frac{\text{a}\times\sin\alpha_2}{d_2\times\sin 1''} = sin^{-1}\frac{150\times\sin 63^\circ\ 47'\ 45''}{54070\times\sin 1''} = 513.41''$$

$$\theta = \widehat{ATB} = \alpha_1' - \alpha_2' = 71^\circ\ 50'\ 59.72''$$

Satellite Station – Numerical Exercise

➢ (2) Instead of a main triangulation station A which is not accessible, a theodolite was setup at station S 10.44 ft apart and approximately south-east from A. The observed direction to A was $43^\circ\ 22'\ 15''$ while that to B was $158^\circ\ 48'\ 57''$ and that to C was $227^\circ\ 25'\ 41''$. The lengths of AB and AC are 16560 ft and 21580 ft, respectively. Compute angle BAC.

$$\alpha_1 = directions(SB - SA) = 115^\circ\ 26'\ 42''$$

$$\alpha_2 = 360 - directions(SC - SA) = 175^\circ\ 56'\ 34''$$

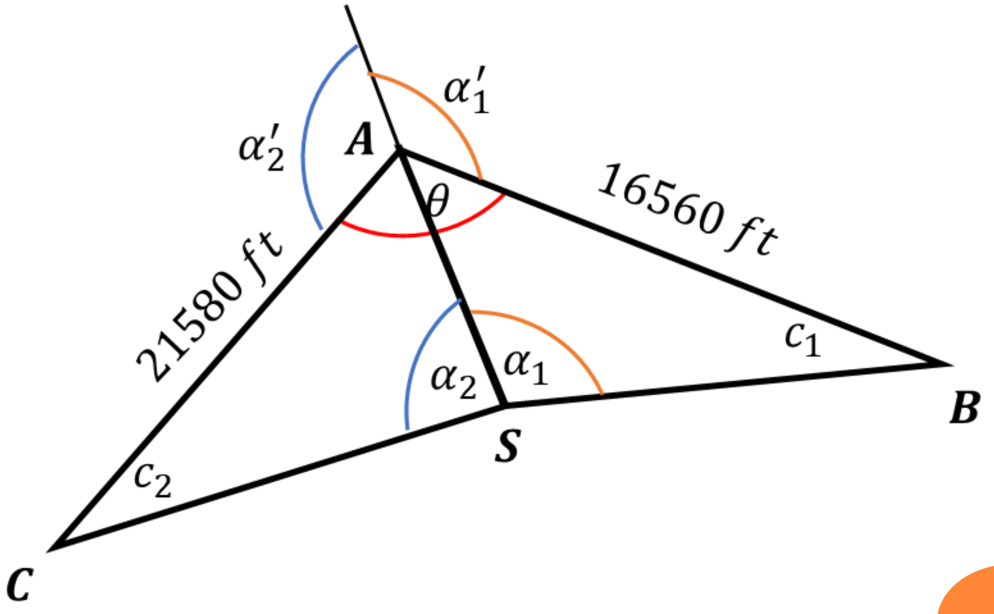
$$c_1 = sin^{-1}\frac{a \times \sin\alpha_1}{AB \times \sin 1''} = sin^{-1}\frac{150 \times \sin 135^\circ\ 42'\ 15''}{71280 \times \sin 1''} = 117.42''$$

$$c_2 = sin^{-1}\frac{a \times \sin\alpha_2}{AC \times \sin 1''} = sin^{-1}\frac{150 \times \sin 63^\circ\ 47'\ 45''}{54070 \times \sin 1''} = 7''$$

$$\alpha_1' = \alpha_1 + c_1$$

$$\alpha_2' = \alpha_2 + c_2$$

$$\theta = \widehat{BAC} = 360 - (\alpha_1' - \alpha_2') = 68^\circ\ 34'\ 39.58''$$



THANK YOU

End of Presentation